Exploring musculoskeletal redundancy using null space projection for evaluation of knee reaction loads

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Summary

It is an undeniable fact that musculoskeletal systems are inherently redundant, i.e., there are more degrees of freedom than those required to perform certain tasks and each degree of freedom is actuated by multiple muscles. This over-availability poses numerous challenges in the process of modeling and simulation that can negatively affect the validity of the models and the obtained results, rendering their application frequently inappropriate for clinical practice. The projection from a low- to a high-dimensional space is not uniquely defined (e.g., muscle sharing problem), however, this indeterminacy can be captured mathematically using the notion of null space. This study presents a method for calculating the feasible muscle forces that satisfy the movement and physiological muscle constraints. Its importance is demonstrated in the context of joint reaction analysis, where it is shown that misinterpretation of the results is possible if the null space solutions are ignored.

Introduction

The muscle forces $\boldsymbol{f}_m \in \mathbb{R}^m$ are related to the generalized forces $\boldsymbol{\tau} \in \mathbb{R}^n$ through the moment arm matrix $\boldsymbol{R}$. When the muscle forces are known, the generalized forces are calculated uniquely. Since the muscle forces cannot be measured non-invasively, they are estimated from experimentally measured kinematics and externally applied forces. However, this approach results in an underdetermined set of equations, which is typically solved by formulating an optimization problem that minimizes some objective criterion. A particular solution will not only bias the results, but will also affect the calculation of other quantities that depend on the muscle forces [1]. Therefore, identification of the feasible solution space can help to properly interpret results obtained from the redundant musculoskeletal systems.

Methods

In a typical experimental setup the motion and externally applied forces are recorded. Given these recordings, inverse kinematics and inverse dynamics are performed in order to assess the model kinematics and kinetics that satisfy the experimental measurements. Instead of estimating the muscle forces by forming an optimization problem, one could find the family of solutions that satisfy the following equation for the known $\boldsymbol{\tau}$

$$\boldsymbol{f}_m = -\boldsymbol{R}^{+T}\boldsymbol{\tau} + \boldsymbol{N_R}\boldsymbol{f}_{m0}, \boldsymbol{N_R} = \boldsymbol{I} - \boldsymbol{R}\boldsymbol{R}^{+} \tag{1}$$

where $\boldsymbol{N_R}$ represents the null space moment arm matrix and $\boldsymbol{f}_{m0}$ an arbitrarily selected vector. Note that $\boldsymbol{f}_m$ spans the entire $\mathbb{R}^m$ for some arbitrary value of $\boldsymbol{\tau}$ and $\boldsymbol{f}_{m0}$, whereas in reality muscle forces are strictly positive (contraction) and bounded (limited force). However, the null space term can provide a suitable correction in order to satisfy the physiological muscle constraints. Assuming a linear muscle model

$$\boldsymbol{f}_m = \boldsymbol{f}_{max} \circ \boldsymbol{a}_m, \boldsymbol{0} \preccurlyeq \boldsymbol{a}_m \preccurlyeq \boldsymbol{1} \tag{2}$$

where $\boldsymbol{f}_{max}$ denotes the muscle strength and $\boldsymbol{a}_m$ the muscle activation, one could observe that Eqs. (1) and (2) must be equal

$$-\boldsymbol{R}^{+T}\boldsymbol{\tau} + \boldsymbol{N_R}\boldsymbol{f}_{m0} = \boldsymbol{f}_{max} \circ \boldsymbol{a}_m, \boldsymbol{0} \preccurlyeq \boldsymbol{a}_m \preccurlyeq \boldsymbol{1}$$

$$\begin{bmatrix} -\boldsymbol{N_R} \\ \boldsymbol{N_R} \end{bmatrix} \boldsymbol{f}_{m0} \preccurlyeq \begin{bmatrix} -\boldsymbol{R}^{+T}\boldsymbol{\tau} \\ \boldsymbol{f}_{max} + \boldsymbol{R}^{+T}\boldsymbol{\tau} \end{bmatrix}. \tag{3}$$

It can be shown that Eq. (3) is convex and bounded thus can be sampled for $\boldsymbol{f}_{m0}$ using vertex enumeration techniques [1]. Therefore, the feasible muscle forces can be obtained by adding the solutions that satisfy Eq. (3) to the particular solution

$$\{\boldsymbol{f}_m\} = \{-\boldsymbol{R}^{+T}\boldsymbol{\tau} + \boldsymbol{N_R}\boldsymbol{f}_{m0}^{i}, \forall i\}.$$

Results and Discussion

The importance of evaluating the feasible muscle forces is demonstrated in the context of joint reaction analysis. An accurate estimation of the muscle forces is essential for the assessment of joint reaction loads. The normalized (with respect to body weight) reaction forces on the knee joint during walking are reported along with the heel strike and toe-off events (Figure 1). The shaded area depicts the reaction force range as attributed to the null space solutions of muscle forces. The red dotted line represents the results obtained from OpenSim. The large range of possible values confirms that misinterpretation of results is possible if the null space solutions are ignored. Consequently, the null space contributions can significantly alter the reaction loads without affecting the movement.

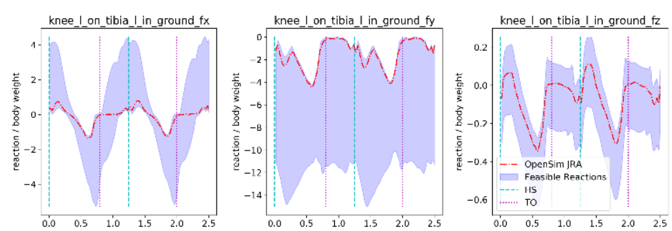

Figure 1: Comparison between feasible reaction forces on the knee (proposed method) and optimization-based obtained (OpenSim).

Conclusions

Null space solutions, although typically ignored in musculoskeletal system simulations, offer deep insights and provide a much broader framework for modelling, simulation and analysis of redundant musculoskeletal systems.

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References

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